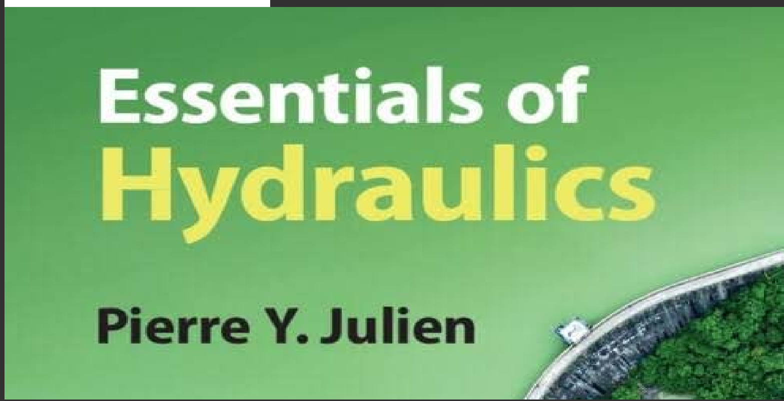
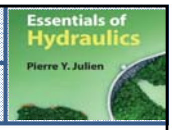
연세대학교
YONSEI UNIVERSITY

3. Hydrodynamics



3. Hydrodynamics | 3.1 Hydrodynamic Force on a Plate



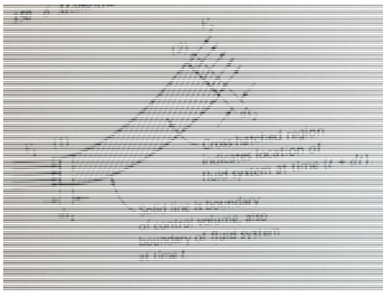
- Hydrodynamics
  - Hydrodynamics is the physics that deals with fluid motion, forces on bodies immersed in fluids, and the motion of a body relative to the motion of fluids.
  - The fundamental principles governing hydrodynamics are the laws of conservation of mass, momentum, and energy.

- Momentum Equation(s)
  - Newton's 2-nd law

$$\sum \vec{F} = m\vec{a}$$

  - Application of this law to the streamtube, which is not straightforward, results in

$$\sum F_i = \rho Q (V_{2i} - V_{1i})$$



- External Forces
  - Surface force: the force that acts across an internal or external surface element in a material body. The surface force can be decomposed into normal forces and shear forces.
  - Body force: the force that acts throughout the volume of a body. Forces due to gravity, electric fields and magnetic fields are examples of body forces.

3. Hydrodynamics | 3.1 Hydrodynamic Force on a Plate

3.1.1 Hydrodynamic Force on a Stationary Plate

- Force from water jet on a stationary plate (x-y 2d problem)
  - In x-direction,

$$\sum F_x = \rho Q(V_{2x} - V_{1x})$$

$$-F_x = \rho Q(V\cos\theta - V)$$

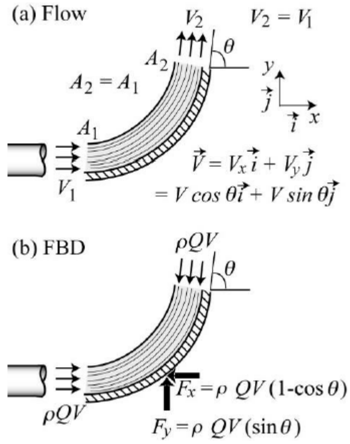
  - In y-direction,

$$\sum F_y = \rho Q(V_{2y} - V_{1y})$$

$$F_y = \rho Q(V\sin\theta - 0)$$

  - Therefore, the force is

$$F = \left(F_x^2 + F_y^2\right)^{1/2}$$



3. Hydrodynamics | 3.1 Hydrodynamic Force on a Plate

3.1.2 Force and Power on a Moving Plate

- Force on a non-moving plate ($v_p = 0$)
  - In x-direction,

$$\sum F_x = \rho Q(V_{2x} - V_{1x})$$

$$-F_x = \rho Q(0 - V)$$

  - In y-direction,

$$\sum F_y = \rho Q(V_{2y} - V_{1y})$$

$$0 = \rho Q/2(V - V) - \rho Q(0)$$

  - Therefore, the force is

$$F = F_x = \rho QV$$

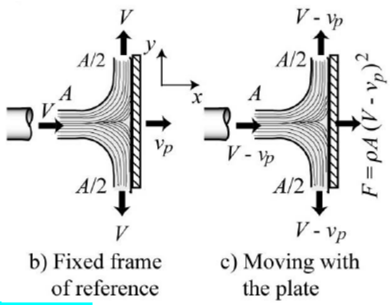


Figure 3.3. Moving vertical plate

- **Moving vane: Moving control volume**

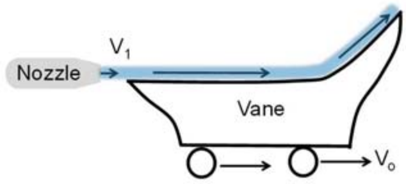
A vane on wheels is considered a moving control volume. It moves with a constant velocity V_o. A stream of water with a nozzle exit velocity V_1 is angled by the vane. What is the relative velocity of the water entering the control volume?

A. $V_1 - V_o$

B. $V_o - V_1$

C. $V_o + V_1$

D. V_1



National Science Foundation, Shell, CU-EEF & Department of Chemical and Biological Engineering, University of Colorado Boulder

- **Force on a moving plate ($v_p > 0$)**
  - $Q = A(V - v_p)$
  - In x-direction,

$$\sum F_x = \rho Q (V_{2x} - V_{1x})$$

$$-F_x = \rho Q\big(0 - (V - v_p)\big)$$

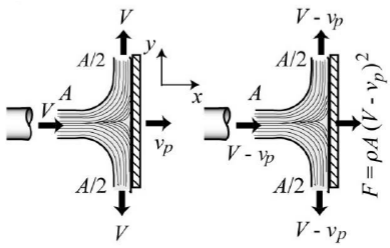
  - In y-direction,

$$\sum F_y = \rho Q (V_{2y} - V_{1y})$$

$$0 = \rho Q\big((V/2 - V/2) - 0\big)$$

  - Therefore, the force is

$$F = F_x = \rho A (V - v_p)^2$$



b) Fixed frame of reference
c) Moving with the plate

Figure 3.3. Moving vertical plate

3. Hydrodynamics | 3.1 Hydrodynamic Force on a Plate

3.1.2 Force and Power on a Moving Plate

Essentials of Hydraulics
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- Power of the moving plate
  - The plate will gain energy that is the force times the displacement of the plate.
  - Or the power is exerted on the plate is the product of the force and the velocity of the plate.

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3. Hydrodynamics | 3.1 Hydrodynamic Force on a Plate

3.1.2 Force and Power on a Moving Plate

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- Ex. 3.1: Force on a moving plate
  - A 10 cfs water jet hits a vertical plate moving at 10 ft/s.
  - The jet velocity is 100 ft/s and the area of the jet is 0.1 ft^2.
  - Find the force and power imparted on the moving plate.

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- Force on a Pipe Bend (x-z 2d problem)

$$\sum F_i = \rho Q(V_{2i} - V_{1i})$$

- In vector notation,

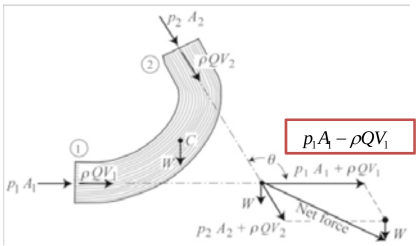
$$\sum \vec{F} = \rho Q(\vec{V_2} - \vec{V_1})$$

- The LHS is

$$\sum \vec{F} = \vec{P_1} + \vec{P_2} + \vec{W} - \vec{R}$$

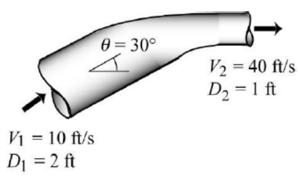
- Therefore, we have

$$\vec{R} = \vec{P_1} + \vec{P_2} + \vec{W} - \rho Q\vec{V_2} + \rho Q\vec{V_1}$$



3. Hydrodynamics | 3.2 Hydrodynamic Force on a Pipe Bend | Essentials of Hydraulics Pierre Y. Julien

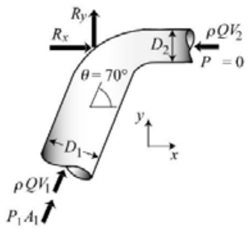
- Ex.3.3: Energy conservation applied to pipe bend
- The horizontal pipe discharges water into atmosphere.
- What is p_1?
- 2d (x-y) problem and $p_2 = 0$



3. Hydrodynamics	3.2 Hydrodynamic Force on a Pipe Bend	Essentials of Hydraulics Pierre Y. Julien

- Ex.3.4: Force to anchor a pipe bend

- What is the horizontal force that anchors this pipe bend in place?



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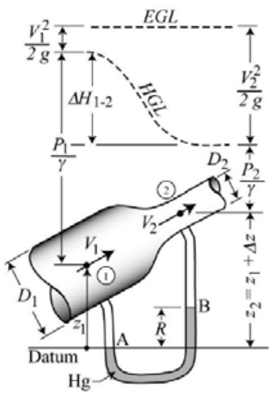
3. Hydrodynamics	3.3 Flow Meters	Essentials of Hydraulics Pierre Y. Julien
3.3.1 Venturi Meter		

- In the late 18th century, the Italian Giovanni Venturi designed a pipe flow meter.
- The principle that the velocity increases after the contraction, is used. Thus measuring the pressure drop **R** enables one to estimate the discharge.



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3. Hydrodynamics | 3.3 Flow Meters

3.3.1 Venturi Meter

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- Energy equation

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

- Manometer reading

$$p_1+\gamma z_1=p_2+G\gamma R+\gamma(z_2-R)$$

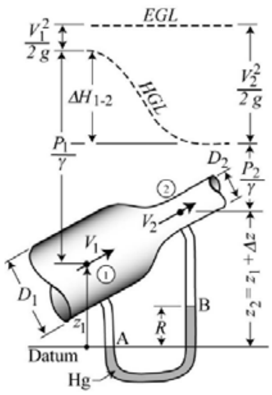
- From Energy equation,

$$\frac{p_1}{\gamma}-\frac{p_2}{\gamma}=\frac{V_2^2}{2g}-\frac{V_1^2}{2g}+(z_2-z_1)$$

$$R(G-1)=\frac{V_2^2}{2g}\left[1-\left(\frac{D_2}{D_1}\right)^4\right]$$

$$V_2=C_G\sqrt{2gR(G-1)}$$

$$Q=C_V C_G A_2\sqrt{2gR(G-1)}$$



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3. Hydrodynamics | 3.3 Flow Meters

3.3.1 Venturi Meter

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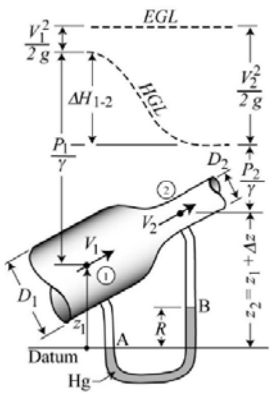
- Discharge

$$Q=C_V C_G A_2\sqrt{2gR(G-1)}$$

- Here

$$C_G=1/\sqrt{1-\left(\frac{D_2}{D_1}\right)^4}$$

- and C_V accounts for energy losses.



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3. Hydrodynamics | 3.3 Flow Meters

3.3.1 Venturi Meter

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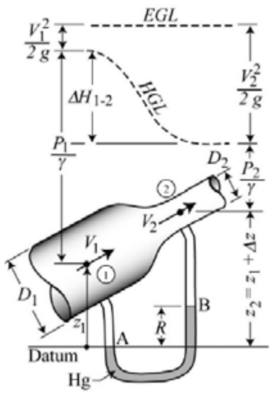
- Ex.3.6: Venturi meter
  - Find the velocity and discharge
  - if D_1 = 1 ft, D_2 = 0.5 ft with R = 1 ft in a mercury manometer.



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3. Hydrodynamics | 3.3 Flow Meters

3.3.1 Venturi Meter

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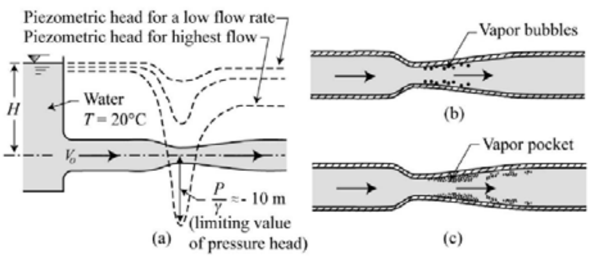
- Cavitation

- At high flows, the piezometric head (or HGL) falls below the pipe elevation and cause negative pressure.

- When reaching the vapor pressure, vapor bubbles form and their collapse can cause cavitation.



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3.3.1 Venturi Meter

• Glen Dam Spillway Tunnel Cavitation May 1983

https://www.reddit.com/r/CatastrophicFailure/comments/riffsm/glen_dam_spillway_tunnel_cavitation_may_1983/

3.3.2 Flow Nozzle and Orifice

• The analysis of orifices is very similar to that of the Venturi.

• A flow nozzle has a profiled contraction in a pipe compared to the short and sharp-edged orifice.

$$Q = CA_2\sqrt{2gR(G-1)} \qquad (3.10)$$

• where $C = C_G C_V$. Note that energy loss is noticeable compared to venturi meter due to abrupt contraction.

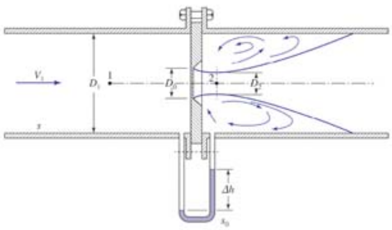
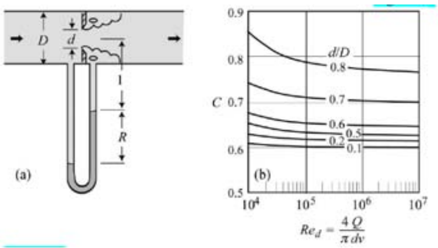


Figure 7.3 Orifice meter

- **Ex.3.8: Orifice meter**
  - Find the discharge
  - if D = 30 cm, d = 15 cm, and R = 20 cm in a mercury manometer.



3. Hydrodynamics | 3.3 Flow Meters
3.3.2 Flow Nozzle and Orifice
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- **Pitot tube**

The French engineer Henri Pitot developed an instrument to measure the fluid flow velocity in the early 18th century. As sketched in Fig. E-3.9, the instrument has two tubes inside one another. The interior tube is facing the flow direction and measures both the pressure and the velocity head, or EGL. The external tube has openings along the side in the direction perpendicular to the flow. The external tube therefore measures the pressure only, i.e. the HGL. The difference in pressure between the two tubes (EGL-HGL) is the velocity head. For instance, if the pressure difference in water is 500 Pa, what is the velocity head?
Solution:

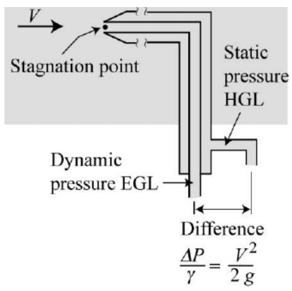

Fig. E-3.9. Pitot tube

$$\Delta p = 500 \text{ Pa}, \text{ and } \frac{\Delta p}{\gamma} = \frac{V^2}{2g}, \text{ thus } V = \sqrt{2\Delta p/\rho} = \sqrt{2 \times 500/1000} = 1 \text{ m/s}.$$

3. Hydrodynamics | 3.3 Flow Meters

3.3.2 Flow Nozzle and Orifice

- Pitot tube

3. Hydrodynamics

- Homework Assignment (Due: one week from today)

- Solve all problems